GOLD MEDAL PROBLEMS

Dear Students,

Gold Medal Problems present you with an opportunity to investigate complex, interesting problems over several days. The purpose is to focus on the process of solving complex problems. You will be evaluated on your ability to show, explain, and justify your work and thoughts.

Completion of a Gold Medal Problem includes 4 parts:

1. *Problem Restatement: State the problem clearly in your own words so that anyone reading your paper will understand the problem you intend to solve.*

2. *Process & Solutions: Describe in detail your thinking and reasoning as you worked from start to finish. Explain your solution and how you know it is correct. Add diagrams when it helps your explanation. Include things that did NOT work, in order to show any changes you had to make along the way. If you can not complete the problem, describe how far you were able to go, what you DO know, and why you are stuck (partial credit may be awarded if your thinking is explained, even if you did not arrive at a correct final solution).*

3. *Reflection: Reflect on your learning and your reaction to the problem. What mathematics did you learn from it? What did you learn about your math problem-solving strategies? Is this problem similar to any other problems you have done before? If yes, how?*

4. *Work & Final Product: Save and submit all of your work, notes, and calculations, even scratch work. This is important because it is a record of your thinking. Do not throw anything away. The work you are using to support your solution must be neat and organized.*

UP TO 5 EXTRA CREDIT POINTS IN THE "HOMEWORK" CATEGORY MAY BE AWARDED.

GM-17. HAPPY NUMBERS

Some numbers have special qualities that earn them a title, such as "Square Number" or "Prime Number." This problem will explore another type of number, called "Happy Numbers."

The number 23 is a Happy Number. To determine if a number is a Happy Number, square each of its digits and add.

$2^2 + 3^2 = 13$

Repeat this process.

$1^2 + 3^2 = 10$

When the final answer is 1, the original number is called a **"Happy Number."**

$1^2 + 0^2 = 1$

The number 34 is not Happy Number, as demonstrated below:

1) $3^2 + 4^2 = 25$
2) $2^2 + 5^2 = 29$
3) $2^2 + 9^2 = 85$
4) $8^2 + 5^2 = 89$
5) $8^2 + 9^2 = 145$
6) $1^2 + 4^2 + 5^2 = 42$
7) $4^2 + 2^2 = 20$
8) $2^2 + 0^2 = 4$
9) $4^2 + 0^2 = 16$
10) $1^2 + 6^2 = 37$
11) $3^2 + 7^2 = 58$
12) $5^2 + 8^2 = 89$

Since 89 is repeated in this series, the "Happy Number" process is in a never-ending loop and, consequently, will never equal 1. Therefore, 34 is not a Happy Number.

Your Task:

- There are 17 two digit "Happy Numbers." Find as many as you can. Describe your technique for finding happy numbers.
- Remember to keep all your work and ideas so you can refer back to them when writing up what you discovered. It will save you time and help you look for patterns if you keep an organized record of what you try.
- Find five 3 digit happy numbers.
- Find five 4 digit happy numbers.